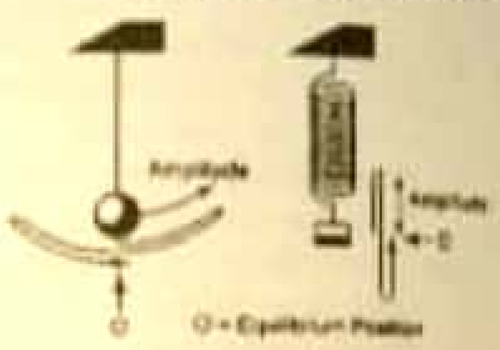
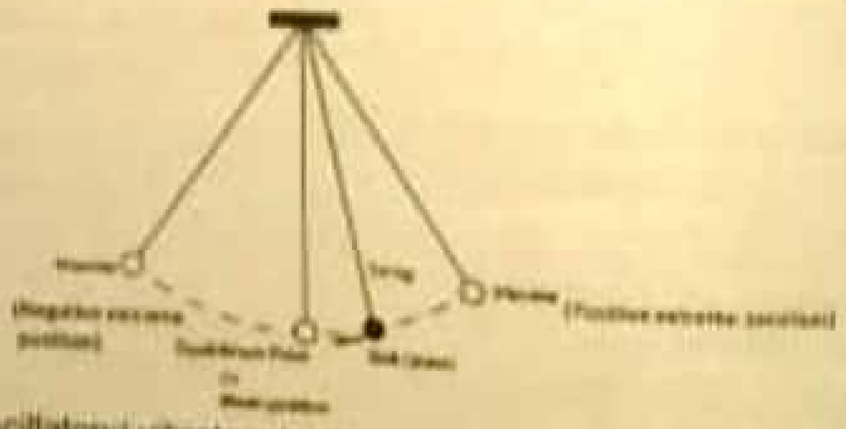
Chapter 7 OSCILLATION

Oscillation:



- Oscillation is the movement of a body from its resting place to the maximum distance it can cover on one side to the maximum distance on the other side and back to its resting place.
- The maximum distance that the body covers on either side of its resting place is equal from its resting place and is called the ***amplitude*** or ***maximum displacement***. The resting point of the body is known as its ***equilibrium state***.
- Oscillation has a definite time period which means the body takes a definite time to complete one oscillation and will remain same for all oscillations if there is no outer force working on the system. It is known as periodic movement and time taken to complete one oscillation is its frequency.
- The to and fro motion of a body is called ***oscillation***.
- If a body repeat its motion about equilibrium position
- **Oscillation** is the repetitive variation, typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.
- Familiar examples of oscillation include a swinging pendulum and alternating current (2nd year chapter 5).
- the beating of the human heart (for respiration) is also an oscillation
- Our lungs also oscillate
- We hear because our ear drums vibrate
- Light waves permit us to see
- Sound waves allow us to hear
- We cannot utter 'vibration' without the oscillation of larynges and vocal cords.

Equilibrium position/mean position:



- The rest position of oscillatory/ vibratory body
- When the mass moves closer to the **equilibrium position**, the restoring force decreases. At the **equilibrium position**, the net restoring force vanishes.

- So at equilibrium or mean position there is not restoring force acts on a body in its original states it mean that acceleration is zero; that's why this state is called equilibrium position
- A **mean position** is any **position** that is a moderate **position** between two other extreme **positions**. It is understood as a middle-of-the-road **position** or compromise between two extreme views.

Vibration:

- Vibration of a body is the movement of the body about its mean position and can be linear, circular, periodic or non-periodic
- When an external force is applied to a body its atoms get displaced from their mean position and due to their binding force tend to come back to their mean position and due to the momentum, they have gained during the motion tend to travel past their mean position to the opposite side and these phenomena continues till friction exhausts all their energy to stop
- This movement of atoms cause vibration of a body and vibration stops when atoms come to halt at mean position.
- Application of vibrations is used in all the music instruments that are played by plucking strings. Loudspeakers work due to the vibrating diaphragm and we hear sound due to vibration of ear drums. Vibration that causes noise can be very harmful for us. Mechanical vibration causes fast wear and tear of machinery parts.

Difference between oscillation and vibration

Oscillation	Vibration
Oscillation is the definite displacement of a body in terms of distance or time	vibration is the movement brought about in a body due to oscillation.
Oscillation takes place in physical, biological systems and often in our society	vibrations is associated with mechanical systems only
Oscillation of a body dissipates energy due to friction which slows it down and ultimately ends the movement but it can be converted to a continuous one by applying external force.	Vibrations tend to end after the dissipation of all the energy of the atoms
Oscillation of a body is used to study the nature of movement and calculations of different energy levels during movement	vibration has a varied range of applications in the industry
Oscillation is about a single body	vibration is the result of collective oscillation of atoms
Every oscillation is vibration	every vibration is not oscillation
If there is restoring force there will be oscillation otherwise no oscillation	

Restoring force:

- Force which compels a body or system to come back to restore position
- The force which tends or force the body toward their normal state/natural state/equilibrium state position
- Oscillation is caused by restoring force

Elastic deformation:

- The temporary deformation in a body is called elastic deformation

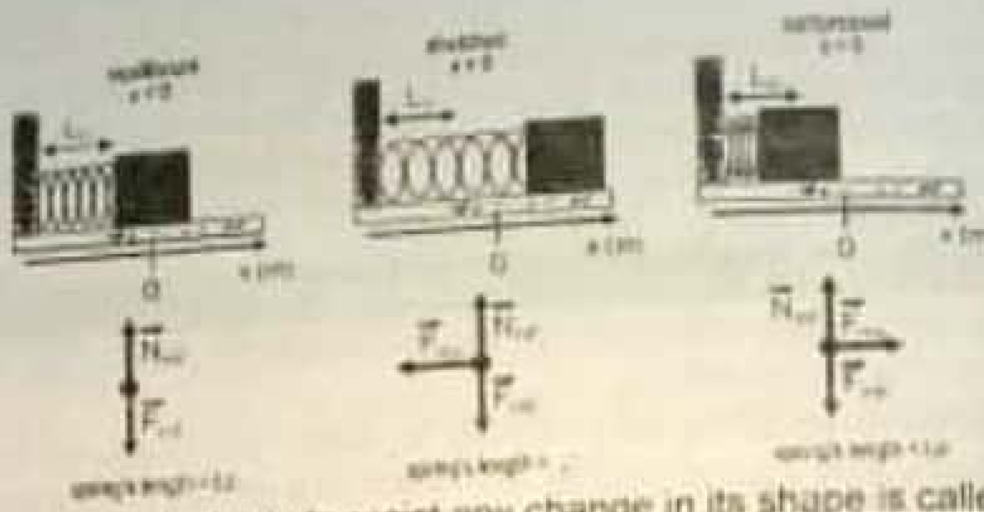
Plastic deformation:

- The permanent deformation in a body is called plastic deformation

Deformation:

- The change in shape or volume

Elasticity:



- The opposing property of a body to resist any change in its shape is called elasticity
- Restoring force causes elasticity
- As you can see the above diagram that at first the spring was at equilibrium position. When we applied a force towards right it stretched, and when we applied a force towards left it compresses. This is example of oscillatory motion and this is due to elastic nature

(***Restoring force ∝ instantenous displacemnet***)

Some basic definition:

i) Instantaneous displacement

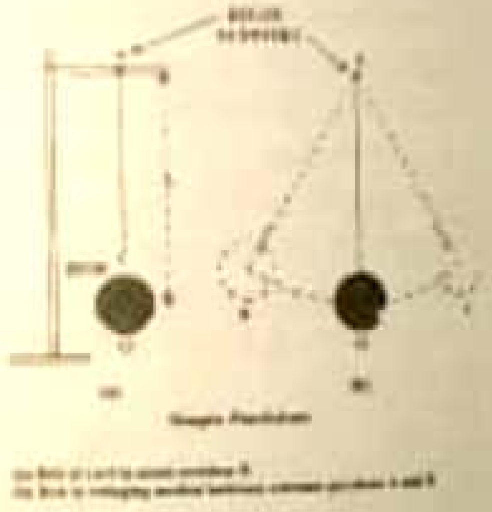
⇒ The displacement of oscillatory body from mean position at any instant of time is called instantaneous displacement.

⇒ It is represented by 'x'

⇒ (i) In an oscillation of a loaded spring, displacement variable is its deviation from the mean position

⇒ (ii) During the propagation of sound wave in air, the displacement variable is the local change in pressure

⇒ (iii) During the propagation of electromagnetic waves, the displacement variables are electric and magnetic fields which vary periodically



ii) Amplitude:

⇒ The maximum displacement of an oscillatory body from its mean position is called amplitude.

⇒ It is generally represented by X_o or X_m

Note that: Every Amplitude can be instantaneous displacement but every instantaneous displacement can't be amplitude.

0 = mean position

A and B → Extreme position

iii) Oscillation

⇒ One complete round trip of an oscillating body is called oscillation.

⇒ In one complete oscillation

⇒ Displacement = 0

⇒ Distance = 4r

iv) Time period:

⇒ The total time required for an osciating body to move to and fro

⇒ It is the time required for an oscillating body to go from mean position→ extreme position, extreme → mean position, mean to negative extreme position, negative extreme → mean position

⇒ It is reciprocal of frequency

v) Frequency:

⇒ The no of vibration per unit time is called frequency

⇒ It is reciprocal of time period

$$f = \frac{n}{t} \rightarrow if\ n = 1\ and\ t = T\ then\ frequency\ is\ given\ by$$

$$f = \frac{1}{t}$$

It is usually measure in a unit: i) Hertz (Hz), ii) Cycle per second

$$1Hz = \frac{1 vibration}{1 second} = \frac{1 vib}{s}$$

vi) Periodic motion:

⇒ A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion.

⇒ And the fixed interval of time after which the motion is repeated is called period of the motion.

⇒ Revolution of earth around the sun (period one year)

⇒ Rotation of earth about its polar axis (period one day)

⇒ Motion of hour's hand of a clock (period 12-hour)

⇒ Motion of minute's hand of a clock (period 1-hour)

⇒ Motion of second's hand of a clock (period 1-minute)

⇒ Motion of moon around the earth (period 27.3 days)

vii) Phase:

⇒ Phase of a vibrating particle at any instant is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

⇒ In oscillatory motion the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalized equation of motion of the vibrating particle

⇒ $y = a \sin\theta = a \sin(\omega t + \varphi_o)$ Here $\theta = \omega t + \varphi_o$ = phase of vibrating particle

Initial phase or epoch: It is the phase of a vibrating particle at t = 0

In $\theta = \omega t + \varphi_o$, when t=0, $\theta = \varphi_o$, here φ_o is the angle of epoch

Same phase: Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of λ or path difference is an even multiple of $(\frac{\lambda}{2})$ or time interval is an even multiple of because 1 time period is equivalent to 2 π rad or 1 wave length (λ)

Opposite phase: When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180°

Opposite phase means the phase difference between the particles is an odd multiple of π (says $3\pi, 5\pi, 7\pi$ ……) or the path difference is an odd multiple of λ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}$ ……) or the time interval is an odd multiple of (T/2)

Phase difference: If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \varphi_1)\ and\ y_2 = a\sin(\omega t + \varphi_2)$$

$$then\ phase\ difference: \Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$$

Simple Harmonic Motion



A particle repeatedly oscillates left and right along an x axis between extreme points x_m and $-x_m$

A.k works

⇒ It is an important type of an oscillatory motion

⇒ A particle is said to be executing simple harmonic motion if it moves to and fro about the mean position or about equilibrium position in a straight line and the graph between displacement of particles from equilibrium position and time is a sine curve. (cosine curve)

⇒ In simple harmonic motion the oscillatory motion f the particles in a straight line such that its displacement from the mean position varies sinusoid ally with time.

⇒ It is periodic motion.

⇒ Why it is called simple harmonic? It so called because it is simplest type of periodic or harmonic motion.

Analysis of simple harmonic motion:

1) **Frequency:** The frequency f of the oscillation is the number of times per second that it completes a full oscillation (a cycle) and has the unit of hertz (abbreviated Hz).
where 1 hertz = 1 Hz = 1 oscillation per second = $1s^{-1}$

2) **Time period:** The time for one full cycle is the period T of the oscillation, which is

$$T = \frac{1}{f} \text{ so } T = \frac{2\pi}{\omega}$$

We know that

$$A = \omega^2 y \rightarrow \omega = \sqrt{A/y}$$

So

$$\text{Time period}, T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{y}{A}} = 2\pi\sqrt{\frac{displacement}{acceleration}}$$

$$frequency, f = f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{acceleration}{displacement}}$$

Frequency and time period of simple harmonic motion does not depend upon the amplitude of the motion.

3) **Displacement:** Displacement x of a particle from the equilibrium position varies with time (t), according to the relation

$$x = x_0 \cos\theta \rightarrow x = x_0 \cos(\omega t) \rightarrow x = x_0 \cos\left(\frac{2\pi}{T}\right)t$$

$$x = x_0 \cos(2\pi f t)$$

Where x_0 amplitude, x = instantenous time

As $\omega = \theta/t$
$t\theta = \omega$

Some important cases:

i. $\theta = 0^0 \Rightarrow \cos 0^0 = 1$, in this case $x = x_0$ and the oscilator is at extreme position
ii. $\theta = 90^0 \Rightarrow \cos 90^0 = 0$, in this case $x = 0 \Rightarrow$ oscilator at mean position.
iii. $\theta = 180^0 \Rightarrow \cos 180^0 = -1$, in this case $x = -x_0 \Rightarrow$ oscilator at opposite extreme position
iv. $\theta = 360^0 \Rightarrow \cos 360^0 = -1 \Rightarrow$ here $x = -x_0 \Rightarrow$ oscilator at opposite extreme position.
v. $\theta = 270^0 \rightarrow \frac{3\pi}{2} \rightarrow \cos 270^0 = 0 \Rightarrow x = 0$ Where and ω or T are constant of the motion

MCQ:

If $x = 12\cos 2T$ cm
Then what will be $x_0 = ?$, $\omega = ?$, $T = ?$, $f = ?$
Solution:

$$x_0 = 12\,cm \rightarrow \omega = 4\,rad/sec$$

$$frequency, f = \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow f = \frac{2}{2\pi} = 1/\pi$$

$$\text{Time period} = \pi\,sec$$

4) **Amplitude:** As we know that the $\sin \omega t$ varies between positive half cycle and negative half cycle then displacement x varies between +ve (+a) and –ve (-a) half cycle. The constant a is called the amplitude of a simple harmonic motion. It is simply a maximum displacement from mean position to extreme position. Or the maximum displacement of the particles executing simple harmonic motion on either side of mean position is called amplitude of the motion. **The amplitude is equal to radius of the reference circle.**

5) **Angular frequency:** The constant ω is called angular frequency of simple harmonic motion and its value depend upon rate of the oscillation.

- ✓ In one cycle the quantity ωt increases by 2π rad while the time t increases by 'T'

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- ✓ We note that time period T is inversely proportional to omega 'ω'. This means that larger the angular frequency smaller will be the time period and more quickly the particle complete a cycle.
- ✓ The simple harmonic motion is caused by "restoring force".
- ✓ Periodic functions are those which represent periodic motion. the sine's and cosines function are simpler periodic motion. A function is said to be periodic if it repeats itself after time period "T".
- ✓ The equation of displacement for simple harmonic motion is $y = a \sin \omega t$
- ✓ If $\varphi = 0^0$ then $y = a \sin \omega t$ this mean at t=0, the particle is at mean position.
- ✓ If $\varphi = 90^\circ$, $y = a\sin(\omega t + 90^0) = a \cos \omega t$. This mean that at t=0 the particle is at the extreme position. Therefore both $y = a \sin \omega t$ and $y = a \cos \omega t$ are valid representation of sinusoidal motion. Motion describe by sine or cosine function of time is known as sinusoidal motion.
- ✓ Sinusoidal motion and simple harmonic motion are same.
- ✓ A particle travelling around a reference circle of radius a at constant angular speed $\omega = 2\pi f$ can be used to represent simple harmonic motion.
- ✓ Because of this relationship between uniform circular motion and simple harmonic motion, we often use the terminology of circular motion in discussing simple harmonic motion.

For example,

$$y = a\sin \omega t = a\sin 2\pi f t = a \sin \frac{2\pi}{T} t$$

$$where\ \omega = 2\pi f = \frac{2\pi}{T}$$

Note that Si unit of f and omega both are hertz.

6) **Phase:** The phase angle of a particle executing simple harmonic motion is its position at t=0. It is also called phase constant. It tells us what the displacement was at t= 0.

$$y = a \sin(\omega t + \varphi)$$

The quantity $(\omega t + \varphi)$ is called the phase of motion, i.e. Displacement of particle in time t and is useful in comparing the motion of two system of particles. The function y is periodic and repeat itself when ωt increases by 2π radian.

7) **Velocity:** The velocity of a particle executing simple harmonic motion at any instant is "The time rate of change of displacement at that instant"

$$instantaneous\ velocity = \omega\sqrt{x_0^2 - x^2}$$

here a represent amplitude and y represent displacement

- ⇒ At mean position x = 0 then $V = \omega x_0$
- ⇒ So we can say that speed is maximum at mean position
- ⇒ At mean position the speed will be equal to the speed of rotatory body
- ⇒ At extreme position $x = x_0$ then $v = 0$

⇒ So velocity of particle executing SHM is maximum when x

⇒ $= 0$, however when the displacement is maximum when $x = x_0$, the velocity of particle is zero

⇒ The maximum value of velocity is called velocity amplitude in simple harmonic motion

i. $v = -v_0 \sin\theta \rightarrow$ *instantenous velocity*

$$v = v_0 \sin\omega t$$

$$v = v_0 \sin\left(\frac{2\pi}{T} t\right)$$

$$v = v_0 \sin(2\pi f t)$$

$$V_0 = \text{maximum velocity/speed}$$

$$v = \text{instantenous velocity/speed}$$

$$-ve \Rightarrow \text{represents direction}$$

Let us now consider some important cases in above equation of instantaneous velocity;

⇒ $v = -v_0 \sin\theta \rightarrow$ if vp = -ve, then oscillator will move from right to left,
so if $\sin\theta = +ve$ then $vp = -ve \rightarrow (0^0 - 180^0)$

⇒ *if* $vp = +ve \rightarrow$ *the oscilator will move from* left to right so $\sin\theta$ is negative (-ve)
→ $(180^0 < \theta < 360^0)$

(i) $v = v_0\sqrt{1-\cos^2\theta}$

(ii) $v = \frac{2\pi}{x_0}\sqrt{x_0^2 - x^2}$

(iii) $v = \omega\sqrt{x_0^2 - x^2} \rightarrow v = \frac{2\pi}{T}\sqrt{x_0^2 - x^2} \rightarrow v = 2\pi f\sqrt{x_0^2 -}$

→ From 0^0 *to* 180^0 all $\sin\theta$ values will be positive

→ $180^0 \rightarrow 360^0$ All $\sin\theta$ values will be negative

⇒ These all are the equation for instantaneous velocity of oscillatc.

> $v = v_0 \sin\theta$ → if we discuss only on the basis of magnitude because magnitude can't be negative. So, we neglect that negative sign and the equation is (as stated above)

MCQ SOLVING STRATEGY:

$$v = \omega\sqrt{x_0^2 - x^2}$$

Case i)

If $x = 0$ (mean position)

$$\Rightarrow v = \omega\sqrt{x_0^2 - 0} \Rightarrow v = \omega\sqrt{x_0^2} \rightarrow v = \omega(x_0)$$

$v = \omega x_0 =$ *maximum velocity, this is velocity at mean position*

$$v = \omega x_0 \rightarrow v_{max} \rightarrow v_0 = \omega x_0 \rightarrow \frac{2\pi}{T} x_0 \rightarrow 2\pi f x_0$$

Case ii)

$$If\ X = \pm x_0 \Rightarrow v = \omega\sqrt{x_0^2 - x_0^2} \Rightarrow v = \omega\sqrt{0} \Rightarrow v = \omega(0)$$

$v = 0 =$ *zero velocity, this is velocity at extreme position*

→ If $\theta = 0^0$ → then $x = x_0$ and $v = 0$ -> oscillator will be at extreme position.

→ If $\theta = 90^0$ then $x = 0$ and $v = v_0$ the oscillator will; be at mean position

MCQ: what will be the velocity of SHM oscillator where its frequency , f = 10HZ and $x_0 = 0.1m$?

SOLUTION:

$$v = 2\pi f x_0 \rightarrow v = 2(\pi)10 \times 0.1 \rightarrow v = 2\pi\frac{m}{sec} \Rightarrow 6 \cdot 2m/sec$$

IMPORTANT FOR MCQS:

We know that:

$$v = v_0\sqrt{1-\cos^2\theta}$$

$$\Rightarrow \cos\theta = \frac{x}{x_0} \rightarrow v = v_0\sqrt{1-\left(\frac{x}{x_0}\right)^2}$$

So when

$$x = \frac{x_0}{2} \rightarrow then, \quad v = \frac{\sqrt{3}}{2}v_0$$

When

$$x = \frac{\sqrt{3}}{2}x_0 \rightarrow then, \quad v = \frac{v_0}{2}$$

ii **Acceleration:** The acceleration of particle which is covering simple at any instant is defined as "rate of change of velocity at that instant"

$$instantenous\ acceleration = A = -\omega^2 x$$

- The acceleration of this particle's changes with displacement y of the particle
- At mean position $x = 0$ so, $A = 0$
- At extreme position $x = x_0$ so, $A = -\omega^2 x_0$
- So, acceleration is zero when $x = 0$, when displacement of particle is maximum its acceleration is also maximum
- Maximum value is also called acceleration amplitude in SHM
  From above equation it is clear that,

$$a = -\omega^2 x \rightarrow a \propto -x$$

- It shows that acceleration is directly proportional to displacement
- So simple harmonic motion also can be defined as,

"A particle is said to be executing simple harmonic motion if its acceleration at any instant is directly proportional to its displacement from the equilibrium position and is always directed toward the center"

- Its direction (-ve) is opposite to the direction of displacement.

ii) **Instantaneous kinetic energy of an oscillator:**

$$k \cdot e_{ins} = \frac{1}{2}mv^2$$

putting values of v

$$k \cdot e_{ins} = \frac{1}{2}m\left[\omega\sqrt{(x_0^2 - x^2)}\right]^2 \Rightarrow k \cdot e_{ins}$$

$$= \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

or

$$k \cdot e_{ins} = m2\pi^2 f^2(x_0^2 - x^2)$$

or

$$k \cdot e_{ins} = \frac{2\pi^2}{T} m(x_0^2 - x^2)$$

→ **If x = 0 (mean position)**

$$k \cdot e_{ins} = \frac{1}{2}m\omega^2 x_0^2 = k \cdot e_{max}$$

So $k.e = \frac{1}{2}m\omega^2 x_0^2$ (mean position)

→ **If x =**

$\pm x_0$ **(extreme position position)**

$$k \cdot e_{ins} = \frac{1}{2}m\omega^2(x_0^2 - x_0^2)$$

$$k \cdot e_{ins} = \frac{1}{2}m\omega^2(0)$$

So $k \cdot e_{ins} = 0$ (extreme position)

iii) **Restoring force on projection:**

X-component of centripetal force is actually restoring force;

$$F_{res} = F_{external} = F_c\cos\theta$$

$$\theta = 0^0 \rightarrow \cos 0^0 = 1$$

$$F_{restoring} = F_{centripetal}$$

iv) **Instantaneous potential energy of an oscillator:**

⇒ At extreme position force will not be maximum.

⇒ During oscillation $F_f = F_{max} = F\ (Extreme\ position)$

$$F_i = 0N\ (at\ mean\ position)$$

average:

$$\Rightarrow <F> = \frac{F_i + F_f}{2} \Rightarrow <F> = \frac{0 + F}{2} \Rightarrow <F> = F/_2$$

Now work done = <F> x cos θ

$$w = <F> x \quad ;\cos 0 = 1$$

$$w = (\frac{F}{2})x$$

NOW:

$$F_c = mr\omega^2 \Rightarrow F_c = mx\omega^2 \Rightarrow \omega = (\frac{mx\omega^2}{2})x \rightarrow \omega = \frac{1}{2}mx^2\omega^2$$

$$\omega = Potential\ energy = \frac{1}{2}mx^2\omega^2$$

Also

$$p.E = \frac{2\pi^2}{T}mx^2 \rightarrow p.E_{mx} = 2\pi^2 f^2 mx^2$$

SOME MCQS POINTS:

$$p.E = \frac{1}{2}m\omega^2 x^2$$

Case i):

$$If\ x = 0\ (mean\ position)$$

$$p.e = 0 \rightarrow no\ restoring\ force, no\ deformation$$

Case ii):

$$if\ x = \pm x_0\ (extreme\ position)$$

$$p.e = \frac{1}{2}m\omega^2 x_0^2 = p.E_{max}$$

$$p.E_{max} = \frac{1}{2}m\omega^2 x_0^2 \rightarrow maximum\ displacement$$

⇒ So potential energy is maximum at extreme position

Important point in simple harmonic motion:

1) The particle moves to and from about the mean position in a straight line
2) The displacement velocity and acceleration all varies sinusoid ally with the time but are not in phase
3) The acceleration of a particle is proportional to displacement but in | the opposite direction due to negative sign
4) The frequency and time period are independent of amplitude of vibration
5) When $x = \frac{x_0}{\sqrt{2}} = 0.707\,x_0$ and $v = \frac{v_0}{\sqrt{2}} = 0.707\,v_0$ the instantaneous kinetic energy will be equal to instantaneous potential energy
6) There are some angle for which potential energy is equal to kinetic energy these angles are $\theta = 45°, 135°$ after way back occur and angle are $225°, 315°$ → for these angle if kinetic energy is 10 joule the potential energy will also be 10 joule, so we can say that for these angle in oscillation k.e instantaneous is equal in magnitude to p.e instantaneous
7) In one oscillation when kinetic energy become maximum the potential energy will become minimum
8) The relationship between frequency is:

$$f_{k.e\,max} = f_{p.e\,max} = 2f_{oscillator}$$

Total energy of simple harmonic motion:

- When a body is executing simple harmonic motion there is restoring force acting on a body due to which the body restores itself
- Due to this force there is potential energy in a simple harmonic motion
- Also, we know that the body is in motion so it also have some kinetic energy which is due to motion of a body
- During oscillation there, potential energy and kinetic energy is continuously changing in interval of times.
- But the total energy or sum of these two energies is constant if we neglect air friction
- The kinetic energy of a body executing simple harmonic motion is given by:

$$E_p = \frac{1}{2} m\omega^2 x^2$$

- The potential energy of a body executing simple harmonic motion is given by:

$$E_k = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

- The total energy of a body executing simple harmonic motion is given by:

$$E = E_p + E_k = \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

$$E = 2\pi m^2 f^2 x_0^2$$

- We know that mass, omega and 'a' are constant so total energy remain constant all the times
- ***The total energy is***

$$E \propto f^2 \rightarrow \textit{directly proportional to square of frequency}$$

$$E \propto x_0^2 \rightarrow \textit{directly proportional to square of amplitude}$$

$$E \propto m \rightarrow \textit{directly proportional to mass}$$

- At extreme position when displacement is maximum $x = x_0$ there will be potential energy
- While at equilibrium position when $x = 0$ the total energy is in the form of kinetic energy
- The kinetic energy at extreme position is zero. The speed of particles at these positions will also be zero
- The kinetic and potential energy are always positive
- Kinetic energy is maximum in mean position.

Cause of simple harmonic motion:

(a) We have studied simple harmonic motion but up till now we have not studied that what actually cause simple harmonic motion

(b) We know that acceleration of a body executing simple harmonic motion is given by

$$A = -\omega^2 y$$

Here y is the displacement of the particle from its mean position. And the negative sign shows that this displacement always directed toward the mean position. So there is a force that acts on this particle that bring it again to its mean position which is called "Restoring force"

$$Restoring\ force = mass \times acceleration = m \times -\omega^2 y$$

$$F = -m\omega^2 y$$

$$F = -ky$$

Where $k (= m\omega^2)$ is a constant of proportionality called force constant for the system

(c) If a body is subjected to restoring force 'f' that is proportional to its displacement from the mean position (y) and is directed opposite to the direction of displacement

- SO THE EQUATION OF SIMPLE HARMONIC MOTION IS

$$F = -ky$$

Example of simple harmonic motion:

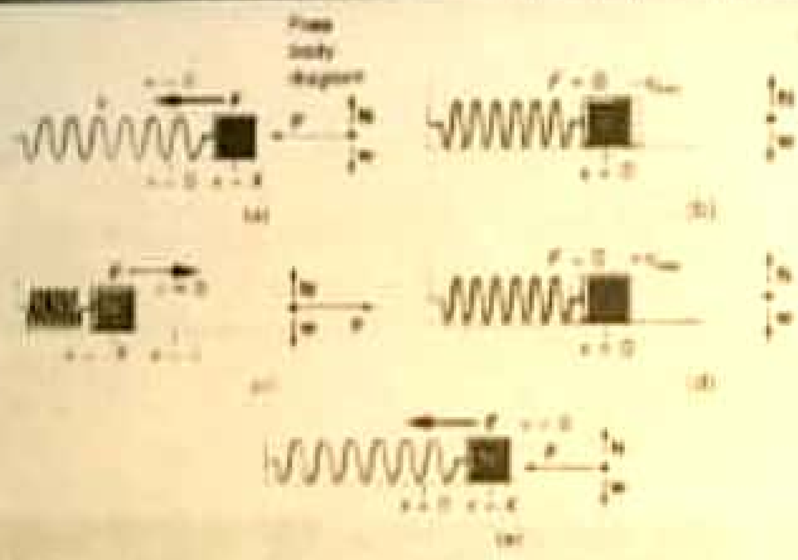
1) Oscillation of a loaded spring

ii) Oscillation of a simple pendulum
iii) Oscillation of liquid in U-tube
iv) Oscillation of a floating cylinder

Summary of some important points in simple harmonic motion:

1) Note that acceleration is directed toward center
2) $a \propto -x \rightarrow a = -(constant)x \rightarrow a = -kx$ so $- - - a = -\omega^2 x$
3) $\omega^2 = constant$, $\omega = constant$
4) $\frac{2\pi}{T} = constant \rightarrow T = constant \rightarrow f = constant$
5) Simple harmonic motion must be periodic motion
6) Simple harmonic motion must be on straight path
7) Every simple harmonic motion is periodic but every periodic motion is not SHM
8) If angle is greater than there will be no SHM but simple harmonic motion must be on straight path greater theta cause the path to be curve
9) At mean position x = 0 so acceleration at mean position is zero
10) If a body is moving in circle (uniform) with constant speed it is SHM(or the projection of a body on a diameter of a circle will perform SHM if a body is in uniform circular motion
11) If non uniform speed then the projection performs vibratory motion

Mass spring system:



An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

⇒ It performs simple harmonic motion
⇒ According to hook's law

$$F_{res} \propto -x \text{ (under elastic limit)}$$

Or we can say that

$$stress \propto strain$$

(-ve sign): the negative sign in above equation shows that restoring force is always directed towards the mean position

As the given equation from hook's law is

$$F_{res} \propto -x$$
$$F_{res} = constant\ (-x)$$
$$F_{res} = kx \rightarrow *$$

➤ In above equation * (k represents stiffness of the spring or it is a spring constant)

➜ The negative sign give us information about the direction of restoring force.

⇒ Note that restoring force is actually responsible here for oscillation

⇒ If we remove the applied force then the net force will be restoring force

i.e

$$F_{net} = F_{res} \Rightarrow ma = -kx$$

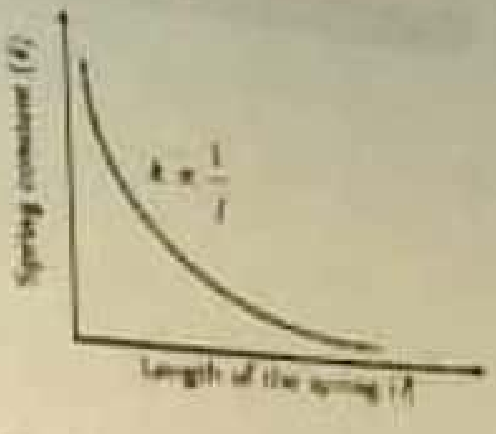
(according to newton second law net force is equal to "mass × acceleration" F = ma)

$$a = -\left(\frac{k}{m}\right)x \rightarrow \text{this is the equation for acceleration}$$

of mass spring system which is same to SHM

Where k is called spring constant

- (i) It's S.I unit Newton/meter C.G.S unit Dyne/cm and dimension is $[MT^{-2}]$
- (ii) Actually k is a measure of the stiffness/softness of the spring
- (iii) For massless spring constant restoring elastic force is same every where
- (iv) When a spring compressed or stretched then work done is stored
- (v) Spring constant depend upon radius and length of the wire used in spring



- (vi) The spring constant k is inversely proportional to the spring length in the form of elastic potential energy in it

$$k \propto \frac{1}{Extension}$$

$$k \propto \frac{1}{Length\ of\ spring}$$

That means if the length of spring is halved then its force constant becomes double.

- (vii) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$. If the constant of a spring is k then spring constant of first part

$$k_1 = \frac{k(n+1)}{n}$$

Spring constant of second part

$$k_2 = (n+1)k$$

and ratio of spring constant

$$\frac{k_1}{k_2} = \frac{1}{n}$$

In case

a = - constant x

where a ∝ - x (which is simple harmonic motion)

- Now compares equation of acceleration for mass spring systen with general equation of acceleration for SHM

As

$$a = -\omega^2 x \rightarrow 1$$

Also

$$a = -({}^{k}/_{m})x \rightarrow 2$$

From the above two equation we can equalize some value.

a. A is equation 1 is same to a(acceleration) in equation 2

b. ω^2 in equation 1 is same or equal to $({}^{k}/_{m})$ in equation 2

So, we can say that

$$\omega^2 = ({}^{k}/_{m})$$

Or

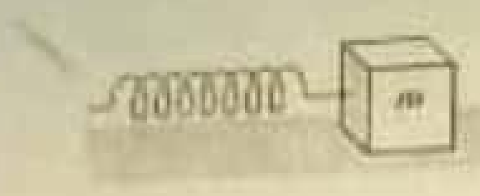
$$\omega = \sqrt{\frac{k}{m}}$$

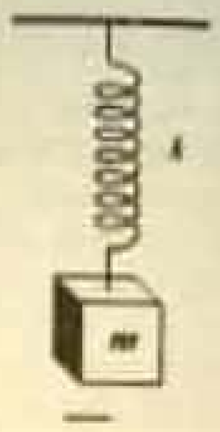
- The above equation shows that

$$\omega \propto \sqrt{k}\ and\ \omega \propto \frac{1}{\sqrt{m}}$$

Spring Pendulum:

- A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring (fig.) constitutes a linear harmonic spring pendulum





$$\text{Time period, } T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{k}}$$

And Frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Because f=1/t

- **(1)** Time period of a spring pendulum depends on the mass suspended

$$T \propto \sqrt{m} \text{ and } f \propto \frac{1}{\sqrt{m}}$$

i.e. greater the mass greater will be the inertia and so lesser will be the frequency of oscillation and greater will be the time period

- **(2)** The time period depends on the force constant k of the spring i.e

$$T \propto \frac{1}{\sqrt{k}} \text{ or } f \propto \sqrt{k}$$

- **(3)** Time of a spring pendulum is independent of acceleration due to gravity. That is why a clock based on spring pendulum will keep proper time everywhere on a hill or moon or in a satellite and time period of a spring pendulum will not change inside a liquid if damping effects are neglected

(4) Massive spring : If the spring has a mass M and mass m is suspended from it, effective mass is given by

$$m_{eff} = m + \frac{M}{3} \text{ Hence}$$

$$T = 2\pi\sqrt{\frac{m_{eff}}{k}}$$

- **(5) Reduced mass** If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m is given by

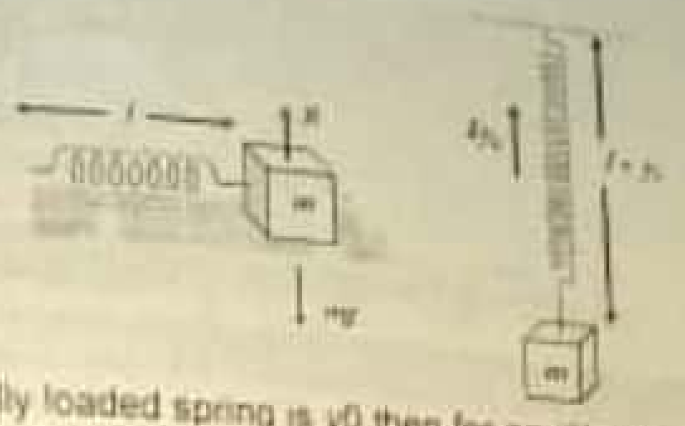
$$\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2} \text{ so that}$$

$$T = 2\pi\sqrt{\frac{m_r}{k}}$$



- (6) If a spring pendulum oscillating in a vertical plane is made to oscillate on a horizontal surface (or on inclined plane) time period will remain unchanged

(7) Equilibrium position for a spring in a horizontal plain is the position of natural length of spring as weight is balanced by reaction. While in case of vertical motion equilibrium position will be $l + y_0$ with $ky_0 = mg$



If the stretch in a vertically loaded spring is y0 then for equilibrium of mass m,

$$ky_0 = mg,\ i.e.\ \frac{m}{k} = \frac{y_0}{g}$$

so that,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

Time period does not depends on 'g' because along with g, yo will also change in such a way that

$$\frac{y_0}{g} = \frac{m}{k}\ \text{remains constant}$$

Some important points from above equations:

a. $T \propto \sqrt{m}$ and the ratio are $\frac{T_1}{T_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$

b. $T \propto \frac{1}{\sqrt{k}}$ and the ratio are $\frac{T_1}{T_2} = \frac{\sqrt{k_2}}{\sqrt{k_1}}$

c. similarly: $f \propto \frac{1}{\sqrt{m}}$ and $f \propto \sqrt{k}$

Some important cases:

- $a = \left(\frac{k}{m}\right)x$, by keeping another parameters constant $a \propto \frac{1}{m}$
- $a = \left(\frac{k}{m}\right)x$, by keeping another parameters constant $a \propto k$

NOTE: As we know that,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Can also be written as

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{x}}\ \text{and}\ T = 2\pi\sqrt{\frac{x}{g}}$$

Proof of $\frac{k}{m} = \frac{g}{x}$:

- Let us we have a vertical mass spring system, we know that the weight of the mass attached to the spring acting downward while the restoring force acting upward
- They are equal in magnitude but opposite in direction.

$$F_{res} = Weight$$

$$kx = mg \to *$$

$$\frac{k}{m} = \frac{g}{x} = \frac{m}{k} = \frac{x}{g}$$

Hence proved

From equation * $k = \frac{w}{x} \to k \propto w$ and $k \propto \frac{1}{x}$

Important points:

So	$F_{ext} = kx$ $k = \frac{F_{ext}}{x}$ $k \propto \frac{1}{x}$ and $k \propto \frac{1}{l}$	$T \propto \frac{1}{\sqrt{k}}$ $k \propto \frac{1}{l}$ $\sqrt{k} \propto \frac{1}{\sqrt{l}}$ and $T \propto \sqrt{l} \rightarrow f \propto \frac{1}{\sqrt{l}}$

Oscillation of Spring Combination

(1) Series combination:

- ✓ If two springs of spring constants K_1 and K_2 are joined in series as shown then



- ✓ In series combination equal forces acts on spring but extension in springs are different
- ✓ Spring constants of combination:

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$$

- ✓ If n springs of different force constant are connected in series having force constant k_1, k_2, k_3 ...... respectively then

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \cdots\cdots$$

- ✓ If all spring have same spring constant then

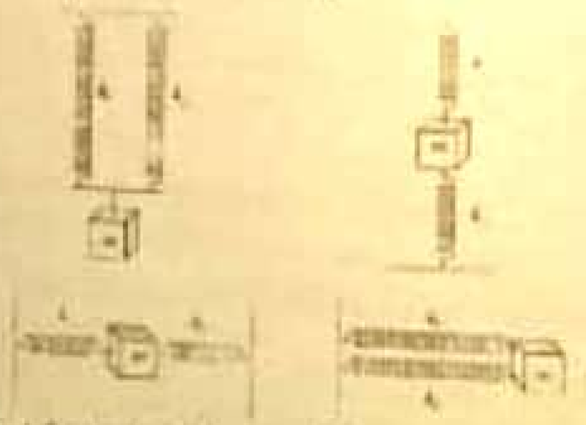
$$k_s = \frac{k}{n}$$

- ✓ Time period of combination:

$$T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

(2) Parallel combination:

- ✓ If the springs are connected in parallel as shown.



- ✓ In parallel combination different forces acts on different springs but extension in springs are same
- ✓ Spring constants of combination $k_P = k_1 + k_2$
- ✓ If n springs of different force constant are connected in parallel having force constant k_1, k_2, k_3 ...... respectively then $k_P = k_1 + k_2 + k_3$ ...

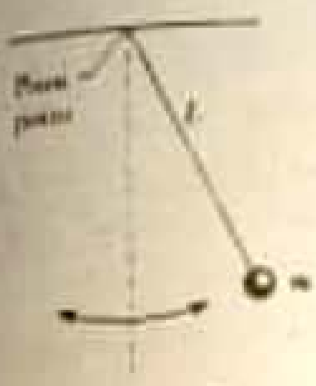
- If all spring have same spring constant then $k_p = nk$
- Time period of combination

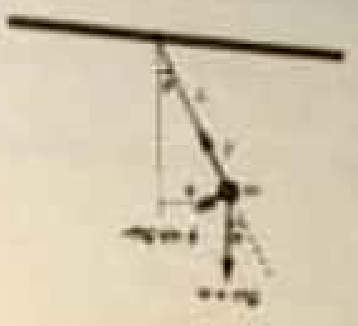
$$T_p = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$$

Simple pendulum:

A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of −mgsinθ toward the equilibrium position—that is, a restoring force.

A simple pendulum





- ***GALELIO*** invented simple pendulum
- A simple pendulum consists of small mass (the pendulum bob) suspended from the end of a light thread. Or A simple pendulum is defined to have an object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string as shown.
- The distance from the point of suspension to the center of gravity of the bob is called the effective length of the pendulum"
- The restoring force also act here which is directly proportional to linear displacement due to this force simple pendulum undergoes simple harmonic motion.
- There are two forces they act on mass. i) tension "T" ii) weight "mg"
- The tension "T" is always directed toward the thread, point of suspension.
- The weight mg is always vertically downwards
- We resolve weight "mg" into its rectangular components, which are $mg\cos\theta$ and mgsin θ
- $mg\cos\theta$ line with the thread and directed opposite to T, the net force $(= T - mg\cos\theta)$ along the thread causes the mass to move circular whose radius is the length of pendulum
- While the component $mg\sin\theta$ act tangent to the arc along the bob moves. It is the component due to which the bob is restored to its mean position and this serves as a restoring force. So, the equation for restoring force become:
- Restoring force $F = -mg\sin\theta$
- Where is the negative sign in above equation shows that the restoring force is always opposite to the direction of displacement.
- We know that when θ is small then $\sin\theta = \theta$ so the equation for restoring force can also be write in the form of $F = -mg\theta$
- Now it's time to give some time to linear displacement, the linear displacement x of the bob from its mean position is related to its angular displacement which is $\theta \rightarrow x = l\theta$
- After calculation we reach to a result that "The acceleration a of the bod is directly proportional to its displacement"
- Note that the direction of acceleration will also opposite to the direction of displacement
- So, we can say that motion of the bob is **simple harmonic motion**

⇒ The string of the simple pendulum should be inextensible although 100% inextensible string is not possible but we should try to take best

⇒ Length of the simple pendulum is equal to length of the string + radius of the bod

$$L_{s.p} = \text{Length of the string} + \text{radius of the bob}$$

⇒ In above figure T represents tension in the string

⇒ Tension in the string is equal to x-component of weight of the bob

⇒ While Restoring force is equal to the y-component of the weight of the bob

⇒ At mean position tension in the string is equal to Weight of the bob which is its maximum value

⇒ If the simple pendulum displaced with small angle motion will be SHM

⇒ During oscillation of simple pendulum tension varies

⇒ At mean position tension is maximum

⇒ At extreme position tension is minimum

Equation for acceleration of simple pendulum is

$a = -g\sin\theta$

so equation of a for simple pendulum is given by,

$$a = -\left(\frac{g}{l}\right)x \rightarrow \text{for simple pendulum}$$

⇒ $a \propto -x$ *equation for the SHM if the angle is small*

Compare equation of acceleration of for simple harmonic motion and equation for simple pendulum

$$a = -\left(\frac{g}{l}\right)x \rightarrow \text{simple pendulum} \rightarrow a = -\omega^2 x \rightarrow SHM$$

$$-\left(\frac{g}{l}\right)x = -\omega^2 x$$

$$\Rightarrow \text{after cutting}$$

$$\Rightarrow \omega^2 = \frac{g}{l} \text{ or } \omega = \sqrt{\frac{g}{l}}$$

$$\Rightarrow \omega^2 = \frac{k}{m} \text{ or } \omega = \sqrt{\frac{k}{m}}$$

i. Time period of simple pendulum:

The time period of a simple pendulum executing simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This show that

$$T \propto \sqrt{l} \text{ and } T \propto \sqrt{g}$$

⇒ It is interesting to hear, which is shown by the equation that the time period of simple pendulum does not depend upon mass and amplitude of the pendulum bob it only depend upon length and value of g. T

⇒ In Muree value of g is less so time period will greater at Muree, while in Peshawar value of g is greater so the time period of simple pendulum will be less

ii. Frequency of simple pendulum:

It is known to us that frequency is inversely related to time period so " the equation for frequency of simple pendulum oscillating with simple harmonic motion is given by"

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

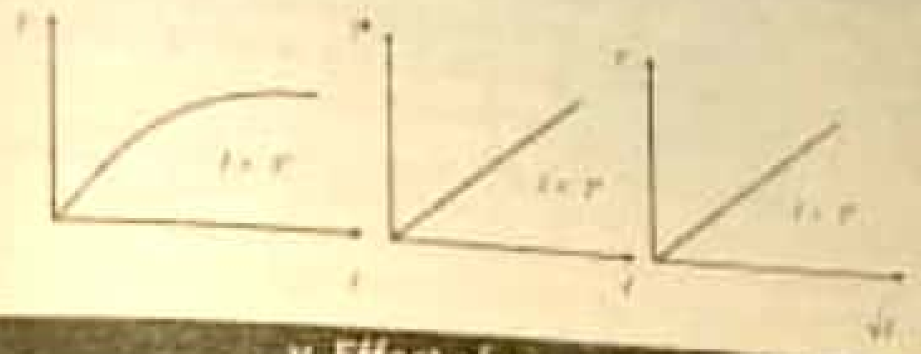
iii. Mass of the bob :

⇒ Time period of simple pendulum is also independent of mass of the bob. This is why

- (i) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.
- (ii) If a girl is swinging in a swing and other sit with her, the time period remains unchanged.

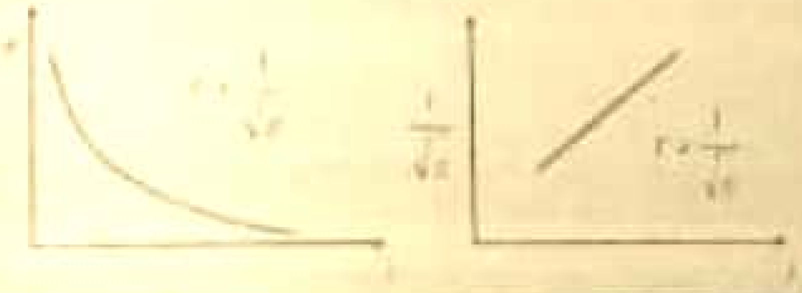
iv. Length of the pendulum:

- Time period $T \propto \sqrt{l}$ where l is the distance between point of suspension and center of mass of bob and is called effective length.
- (i) When a sitting girl on a swinging swing stands up, her center of mass will go up and so l and hence T will decrease.
- (ii) If a hole is made at the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the center of mass will be at the center of the sphere. However, as water drains off the sphere the center of mass of the system will first move down and then will come up. Due to this l and hence T first increase, reaches a maximum and then decreases till it becomes equal to its initial value.
- (iii) Different graphs



v. Effect of g :

- As $T \propto \frac{1}{\sqrt{g}}$ i.e. as g increase T decreases
- (i) As we go high above the earth surface or we go deep inside the mines the value of g decrease, hence time period of pendulum (T) increases.
- (ii) If a clock, based on simple pendulum is taken to hill (or on any other planet), g will decrease so T will increase and clock will become slower.
- (iii) Different graphs



vi. Effect of temperature on time period:

- If the bob of simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

Summary of some interesting facts about simple pendulum:

- It is known form the equations above that time period and frequency of simple pendulum depend upon the length and value of g.
- The fact s provides a simple method to measure 'g'.
- Simple pendulum of known length 'l' is made to oscillate. The frequency 'f' is recorded. Then after that g can easily be calculated.
- On the moon g is about one-sixth that on the earth, so we can say that a pendulum of a given length on the moon would have a period over twice as long as on the earth.

⇒ With the help of equation of time period of simple pendulum height and value of g can be calculated

As;

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So;

$$T^2 = 4\pi^2 l$$

$$g = \frac{4\pi l}{T^2} \rightarrow \text{this equation is used for the calculation of value of } g$$

$$H = \frac{gT^2}{4\pi^2} \text{ (HEIGHT OF BUILDING)}$$

Some important points:

⇒ Omega ω, time period and frequency independent of mass and amplitude of a bob

⇒ Two oscillator 1 attaches to wood bob second attaches to iron bob of the same size then time period and angular frequency and omega will be same for both

⇒ If the word is same mass (not same size); then

$$\text{Let say } m_{wood} \text{ bob} = m_{iron} \text{ bob}$$

So, radius of wood bob is greater than radius of iron bob

length of wood bob is greater the length of the iron bob

Time period of the wood bob will be greater than time period of the iron bob

⇒ If same radius

$$r_i = r_w$$
$$m_i > m_w$$
$$l_i = l_w \text{ (length)}$$
$$T_i = T_w$$

⇒ In one oscillation two-time kinetic energy and two-time potential energy becomes maximum

⇒ If kinetic energy is maximum its potential energy will be minimum

⇒ Simple pendulum filled with fluid during oscillation bob gets leaked then its Time period increases and frequency decrease during leakage but when become empty time period decrease and frequency increase

⇒ If potential energy = 20% of E_{total} then k.e = 80% of E_{total}, because total energy is conserved or equal to 100% always

Pendulum in a lift:

⇒ If the pendulum is suspended from the ceiling of the lift

⇒ **(i) If the lift is at rest or moving down ward /upward with constant velocity**

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ and } f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

⇒ **(ii) If the lift is moving upward with constant acceleration a**

$$T = 2\pi\sqrt{\frac{l}{g+a}} \text{ and } f = \frac{1}{2\pi}\sqrt{\frac{g+a}{l}}$$

Time period decreases and frequency increases

⇒ **(iii) If the lift is moving down ward with constant acceleration a**

$$T = 2\pi\sqrt{\frac{l}{g-a}} \text{ and } f = \frac{1}{2\pi}\sqrt{\frac{g-a}{l}}$$

Time period increase and frequency decreases

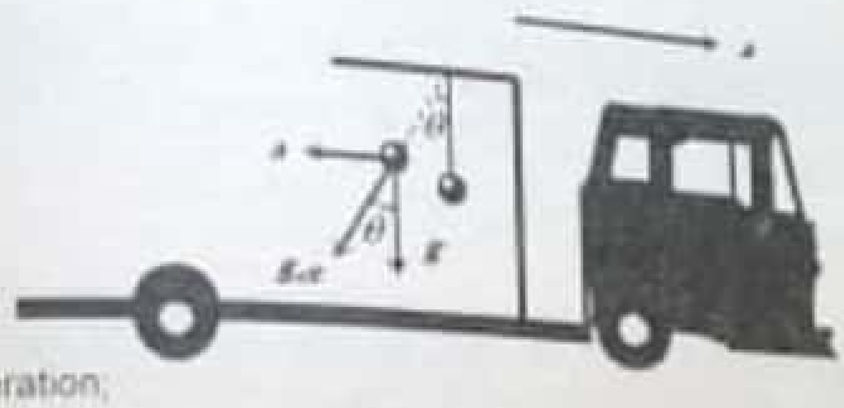
(iii) If the lift is moving down ward with acceleration $a = g$

$$T = 2\pi\sqrt{\frac{l}{g-g}} = \infty \text{ and } f = \frac{1}{2\pi}\sqrt{\frac{g-g}{l}} = 0$$

It means there will be no oscillation in a pendulum. Similar is the case in a satellite and at the centre of earth where effective acceleration becomes zero and pendulum will stop.

Pendulum in accelerated vehicle:

The time period of simple pendulum whose point of suspension moving horizontally with acceleration a



In this case effective acceleration;

$$g_{eff} = \sqrt{g^2 + a^2}$$

$$T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \text{ and } \theta = \tan^{-1}(a/g)$$

Simple pendulum suspended in a car that is moving with constant speed v around a circle of radius r

$$T = 2\pi\frac{\sqrt{l}}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}$$

Some other types of pendulum:

Infinite length pendulum:

If the length of the pendulum is comparable to the radius of earth then;

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

- If $l \ll R$, then $\frac{1}{l} \gg \frac{1}{R}$ so $T = 2\pi\sqrt{\frac{l}{g}}$
- If $l \gg R (\to \infty)$ then $\frac{1}{l} < \frac{1}{R}$

$$So\ T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}} \cong 84.6 \text{ minutes}$$

and it is the maximum time period which an oscillating simple pendulum can have

$$if\ l = R\ so\ T = 2\pi\sqrt{\frac{R}{2g}} \cong 1\ hour$$

Second's pendulum:

A simple pendulum whose time period is two second is called second pendulum

- It is the fact due to which it is called second pendulum because it makes a swing from one side to the other in two 2 seconds
- Let us find the length of second's pendulum which is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$T = 2s, \quad g = \frac{9.8m}{s^2}$$

$$l = \frac{(2)^2 \times 9.8}{4\pi^2} = 0.993m = 99.3cm$$

Hence length of second's pendulum is 99.3 cm or nearly 1 meter on earth surface
The length of the second pendulum depend upon the value of g it changes when the value of g changes. For the moon the length of the second's pendulum will be 1/6 meter

$$As \quad g_{moon} = \frac{g_{earth}}{6}$$

Free and force oscillation:

i) Free oscillation:

- When a body is capable of oscillating is made to vibrate with its own natural frequency is said to be the body execute free oscillation
- So, when a body is capable of oscillating is displaced from its mean position and then left free. It begins to oscillate freely with a definite frequency
- The frequency depends upon intrinsic properties such as that of shape, elastic nature etc
- This frequency is generally called natural frequency
- And the oscillation due to natural frequency are called free oscillation.
- For simple pendulum the natural frequency depends upon its length and value of g
- In the absence of frictional force, the amplitude which is the maximum displacement of free oscillation remains constant. Which is also in case of un-damped oscillation
- But in actual, the amplitude of free oscillation decreases gradually with time and comes to zero due to the frictional forces
- **Example of free oscillations are:** Plucking the string of sitar and then left it

ii) Forced oscillation:

- When a body is maintained in a state of oscillation by an external periodic force of frequency other than natural frequency is called forced oscillation
- The frequency of forced oscillation is equal to the frequency of periodic force.
- The external force that is applied on a body is called driver
- The body set into oscillation by driver is called driven oscillation
- Most of the oscillation occur in different machines are forced oscillation
- They are produced and maintained by external force
- **Example is:** A vibrating violin string stretched tightly between two clamps does not produces intense sound

Resonance:

- When a body is oscillating in a condition such that it oscillates with same frequency as that natural frequency then the condition is called resonant oscillation
- The phenomena of producing resonant oscillation is called resonance
- It is a particular case of forced oscillation
- The natural resonant frequency of a system is called resonant frequency.

- The amplitude with which the body will oscillate must be large
- The relation between natural frequency and resonant frequency is given by:
  $F_r = n f_n$ → where n is integral multiple, f_n is natural frequency, f_r is resonant frequency

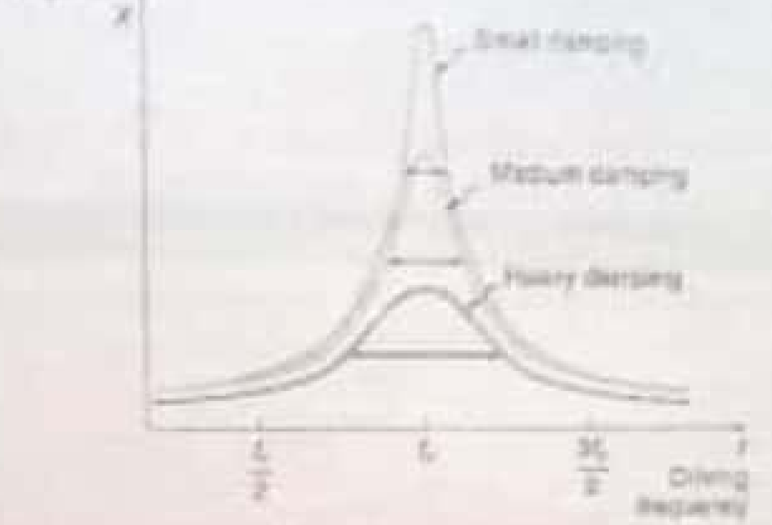
APPLICATION OF RESONANCE

- Microwave oven work on the principle of resonance
- MRI (magnetic resonance imaging) also work on the principle of resonance
- All communication system such as T.v. radio, telephone work on the principal of resonance

Examples of resonance:

- Soldiers are asked to break step while crossing a bridge.
- If the soldiers march in step there is a possibility that the frequency of footsteps may become equal to natural frequency of bridge
- Due to resonance, the bridge may start vibrating violently, thereby damping itself.

Sharpness of resonance:



- We know that at resonance the amplitude is very large when damping is small. So, damping prevents the amplitude from excessively becoming large. The amplitude decreases rapidly at a frequency slightly different from resonant frequency
- So the definition is "if the damping force are small then system will oscillate with large amplitude
- Damped oscillation is also applied in the shock absorber of the car which provides damping force to prevent excessive vibrations.
- Bandwidth is defined as "the range of frequency with which the system resonates
- Here are some important relations

$$Amplitude \propto \frac{1}{Dampind\ force}$$
$$Bandwidth \propto Damping\ forces$$
$$Amplitude \propto \frac{1}{Band\ width}$$
$$sharpness \propto \frac{1}{Band\ width}$$
$$sharpness \propto Amplitude$$
$$Sharpness \propto \frac{1}{Damping\ force}$$

Damped and un-damped oscillation:

i) Damped oscillation:

- The word damped means "diminish"
- The oscillation whose amplitude goes on decreasing with respect to time then these types of oscillation are called damped oscillation

⇒ It is due to some dissipative forces which waste energy of our system due to which system stop oscillation.

⇒ Damping force reduces the velocity and the Kinetic Energy of the moving body

⇒ Damping or dissipative forces generally arises due to the viscosity or friction in the medium and are non-conservative in nature

⇒ In most of the system, especially those which involve frictions associated with slow motion through a viscous fluid the damping force is proportional to velocity and acts opposite to the motion

⇒ These oscillations are forced oscillations

⇒ The force which oppose oscillation due to which energy is lost is known as damping force example. AIR DRAG FORCE. FRICTIONAL FORCE

ii) Un-damped oscillation:

⇒ The oscillation whose amplitude remain constant with time are called un-damped oscillation

⇒ Such oscillation can occur if frictional forces are absent

⇒ Example: if bob of a pendulum is displaced in a vacuum and then release the bob will continue to execute simple harmonic motion of constant amplitude

Some parameters:

⇒ ***Displacement, potential energy and acceleration have same phase, at a time they become maximum and minimum.***

⇒ x and P.E = $\Delta\varphi = 0^\circ$

⇒ P.E and a = $\Delta\varphi = 0^\circ$

⇒ x and a = $\Delta\varphi = 0^\circ$

⇒ ***velocity and kinetic energy have same phase.***

⇒ v and K.E → $\Delta\varphi = 0^\circ$

⇒ ***Between (displacement, potential energy, acceleration) and (velocity kinetic energy) have*** $\Delta\varphi = 90^\circ$

⇒ Between these two sets of variables $\Delta\varphi = 90^\circ$

⇒ ***Between vectoral acceleration and displacement the phase is*** $\Delta\varphi = 180^\circ$ → This is phase difference between direction $\vec{x}$ and $\vec{a}$.

⇒ $\vec{a}$ *always directed towards mean position.*

⇒ $\vec{x}$ *always directed outwards.*

Some important points:

⇒ Between $\sin\theta$ and $\cos\theta$ there is a phase difference of 90° because $x = x_0 \cos\theta$ and $v = v_0 \sin\theta$ and they have $\Delta\varphi = 90^\circ$

⇒ $x = x_0 \cos\theta$

⇒ $v = v_0 \sin\theta$

We will use it when starting position is extreme position.

⇒ $x = x_0 \sin\theta$

⇒ $v = v_0 \cos\theta$

We will use them when starting position is mean position

Formula Sheet

S/N	QUANTITY	EQUATION
1)	*Equation for simple harmonic motion*	$a \propto -x$
2)	*SHM with reference to circle*	Same as above $a \propto -\omega^2 x$
3)	*Instantenous displacement of Oscillator*	$x = x_o \cos\theta$ $x = x_o \cos(\omega t)$ $x = x_o \cos\left(\frac{2\pi t}{T}\right)$ $x = x_o \cos(2\pi f t)$
4)	*Instantaneous velocity of Oscilator*	$v = v_o \sin\theta$ $v = v_o \sin\omega t$ $v = v_o \sin\left(\frac{2\pi t}{T}\right)$ $v = v_o \sin(2\pi f t)$ SOME OTHER EQUATIONS $V = V_o\sqrt{1 - \cos\theta}$ $V = \frac{v_o}{x_o}\sqrt{x_o^2 - x^2}$ $V = v_o\sqrt{1 - \left(\frac{x}{x_o}\right)^2}$ $V = \omega\sqrt{x_o^2 - x^2}$
5)	*Instantaneous K.E of Oscilator*	$K.E_{inst} = \frac{1}{2} m\omega^2 (x_o^2 - x^2)$ $K.E_{inst} = m 2\pi^2 f^2 (x_o^2 - x^2)$ $K.E_{inst} = \frac{2\pi^2}{T} m (x_o^2 - x^2)$
6)	*Instataneous P.E of Oscilator*	$\langle F \rangle = \frac{F}{2}$ $W = P.E = \frac{1}{2} m\omega^2 x^2$ $P.E = \frac{2\pi^2 m}{T^2} x^2$ $P.E_{inst} = 2\pi^2 f^2 m x^2$
7)	*Total Energy of Oscilator (K.E + P.E)*	$E_{inst} = \frac{1}{2} m\omega^2 x_o^2$ $E_T = \frac{2\pi^2}{T^2} m x_o^2$ $E_T = 2\pi^2 f^2 m x_o^2$
8)	*Mass spring system*	$F_{res} = -kx$ $a = -\left(\frac{k}{m}\right)x$ $\omega^2 = \frac{k}{m} \Rightarrow as\ \omega^2 = \frac{g}{x}$ $\omega = \sqrt{\frac{k}{m}}$ Frequency

		$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$ Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$ Instantaneous Energy $E_{inst} = \frac{1}{2}kx_0^2$
9)	*Spring connected in series*	$\frac{1}{k_{net}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} - - - - + \frac{1}{k_n}$
10)	*Spring connected in parallel*	$k_{net} = k_1 + k_2 + k_3 - - - + k_n$
11)	*Simple Pendulum*	$a = -g\sin\theta$ $a = -(\frac{g}{l})x$ $\omega^2 = \frac{k}{m} = \frac{g}{l} = \frac{g}{x}$ Time period $T = 2\pi\sqrt{\frac{l}{g}}$ Frequency $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

Some Important Points and Tricks:

1) All oscillatory motions are necessarily periodic motion but all periodic motion is not oscillatory.
2) Simple pendulum is an example of oscillatory.
3) Simple harmonic motion is a uniform motion.
4) Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic because in simple harmonic motion direction of displacement is also considered.
5) Sine and cosine functions are periodic functions.
6) Sinusoidal functions repeat it values after a definite interval of time.
7) The graph between velocity and displacement for a harmonic oscillator is a parabola.
8) When a simple pendulum is made to oscillate on the surface of moon, its time period increases because Moon is much smaller as compared to earth.
9) Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum.
10) The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.
11) The graph of total energy of a particle in SHM w.r.t. position is a straight line with zero slope.
12) Total energy of particle in SHM remains constant throughout its motion.
13) The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%. It is because Time period is directly proportional to length of pendulum.
14) The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 s^{-1}.

40) If a particle performs S.H.M. whose velocity is v_1 at a x_1 distance from mean position and velocity v_2 at distance x_2

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}, \; T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

$$a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} ; \; x_{max} = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$$

41) If $y_1 = a \sin \omega t$ and $y_2 = b \cos \omega t$ are two S.H.M. then by the superposition of these two S.H.M. we get $y = y_1 + y_2$

42) $y = a \sin \omega t + b \cos \omega t \Rightarrow y = A \sin(\omega t + \varphi)$, this is also equation of S.H.M. where $A = \sqrt{a^2 + b^2}$ and $\varphi = \tan^{-1}(b/a)$

43) In the absence of resistive force the work done by a simple pendulum in one complete oscillation is zero

MULTIPLE CHOICE QUESTIONS

1) The phase of a particle executing simple harmonic motion is $\pi/2$ when it has
(a) Maximum velocity
(b) Maximum acceleration
(c) Maximum energy
(d) Maximum displacement

2) A particle starts S.H.M. from the mean position. Its amplitude is A and time period is T. At the time when its speed is half of the maximum speed, its displacement y is______?
(a) $\frac{A}{2}$ (b) $A/\sqrt{2}$
(c) $\frac{A\sqrt{3}}{2}$ (d) $\frac{2A}{\sqrt{3}}$

3) The amplitude and the periodic time of a S.H.M. are 5cm and 6sec respectively. At a distance of 2.5cm away from the mean position, the phase will be
(a) $5\pi/12$ (b) $\pi/4$
(c) $\pi/3$ (d) $\pi/6$

4) Two equations of two S.H.M. are $y = a \sin(\omega t - \alpha)$ and $y = b \cos(\omega t - \alpha)$. The phase difference between the two is
(a) 0° (b) α° (c) 90° (d) 180°

5) The amplitude and the time period in a S.H.M. is 0.5 cm and 0.4 sec respectively. If the initial phase is $\frac{\pi}{2}$ radian, then the equation of S.H.M. will be
(a) $y = 0.5 \sin 5\pi t$ (b) $y = 0.5 \sin 4\pi t$
(c) $y = 0.5 \sin 2.5\pi t$ (d) $y = 0.5 \cos 5\pi t$

6) The equation of S.H.M. is $y = a \sin(2\pi nt + \alpha)$, then its phase at time t is______?
(a) $2\pi nt$ (b) α (c) $2\pi nt + \alpha$ (d) $2\pi t$

7) A particle is oscillating according to the equation $X = 7 \cos 0.5\pi t$, where t is in second. The point moves from the position of equilibrium to maximum displacement in time______?
(a) 4.0 sec (b) 2.0 sec
(c) 1.0 sec (d) 0.5 sec

8) A simple harmonic oscillator has an amplitude a and time period T. The time required by it to travel from $x = a$ to $x = a/2$ is
(a) T / 6 (b) T / 4
(c) T / 3 (d) T / 2

9) Which of the following expressions represent simple harmonic motion
(a) $x = A \sin(\omega t + \delta)$
(b) $x = B \cos(\omega t + \varphi)$
(c) $x = A \tan(\omega t + \varphi)$
(d) $x = A \sin \omega t \cos \omega t$

10) A 1.00×10^{-20} kg particle is vibrating with simple harmonic motion with a period of 1.00×10^{-5} sec and a maximum speed of 1.00×10^{3} m/s. The maximum displacement of the particle is
(a) 1.59 mm (b) 1.00 m (c) 10 m (d) None of these

11) The phase (at a time t) of a particle in simple harmonic motion tells______?
(a) Only the position of the particle at time t
(b) Only the direction of motion of the particle at time t
(c) Both the position and direction of motion of the particle at time t
(d) Neither the position of the particle nor its direction of motion at time t

12) A particle is moving with constant angular velocity along the circumference of a circle. Which of the following statements is true

15) Damped oscillation indicates loss of energy.

16) In a S.H.M., kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude and in SHM, kinetic energy is zero when potential energy is maximum.

17) If the amplitude of a simple harmonic oscillator is doubled, its total energy becomes four times because the total energy is directly proportional to the square of amplitude of vibration of the harmonic oscillator.

18) For an oscillating simple pendulum, the tension in the string is maximum at the mean position and minimum at the extreme position.

19) The velocity of oscillating bob in simple harmonic motion is maximum at the mean position.

20) The spring constant of a spring is k. When it is divided into n equal parts, then spring constant of one piece is k/n.

21) The spring constant is independent of material used for the spring.

22) The periodic time of a hard spring is less as compared to that of a soft spring.

23) The periodic time depends upon the spring constant, and spring constant is large for hard spring.

24) In extreme position of a particle executing S.H.M., both velocity and acceleration are zero.

25) In S.H.M., acceleration always acts towards mean position.

26) Soldiers are asked to break steps while crossing the bridge because the frequency of marching may be equal to the natural frequency of bridge and may lead to resonance which can break the bridge.

27) The amplitude of oscillation can never be infinite.

28) The energy of oscillator is continuously dissipated.

29) The amplitude of an oscillating pendulum decreases gradually with time because the frequency of the pendulum decreases with time.

30) In simple harmonic motion, the velocity is maximum when acceleration is minimum and Displacement and velocity of S.H.M. differ is phase by $\pi/2$.

31) Suppose a body of mass m vibrate separately with two different springs (of spring constants k_1 and k_2) with time period T_1 and T_2 respectively.

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \text{ and } T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

32) If the same body vibrates with series combination of these two springs then for the system time period

$$T = \sqrt{T_1^2 + T_2^2}$$

33) If the same body vibrates with parallel combination of these two springs then time period of the system

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

34) The pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.

35) If infinite spring with force constant k, 2k, 4k, 8k ______ respectively are connected in series. The effective force constant of the spring will be k/2.

36) Percentage change in time period with l and g. If g is constant and length varies by n%. Then % change in time period,

$$\frac{\Delta T}{T} \times 100 = \frac{n}{2} \times 100$$

37) If l is constant and g varies by n%. Then % change in time period,

$$\frac{\Delta T}{T} \times 100 = -\frac{n}{2} \times 100$$

(Valid only for small percentage change say 5%)

38) Suppose a spring of force constant k oscillates with time period T. If it is divided in to 'n' equal parts then spring constant of each part will become nk and time period of oscillation of each part will become,

$$\frac{T}{\sqrt{n}}$$

39) If these n parts connected in parallel then

$$k_{eff} = n^2 k$$

So time period of the system becomes

$$T' = \frac{T}{n}$$